Causality and Quantum theory

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Motivation of the project

- Certain events may violate causality
- There is a systematic way to verify this
- Quantum mechanical predictions disprove verifications

What is causality?

• Causality denotes a necessary relationship between cause and effect which is the direct consequence of the first.

The envelope experiment



- Blue and red discs are contained in a system of envelopes
- Large envelopes are pre-prepared randomly with the exception that envelopes with same number contain disc of different color

Chain of distribution



Data processing

Example of data

1R	3B
2B	2R
3B	2B

From the data we determine the probabilities of given results.

P(1R,2R) means that Alice chose envelope 1 and found red disc, Bob chose envelope 2 and found red disc, too

The mathematical analysis yields (one of the Bell's inequalities):

P(1R,2R) + P(2R,3R) >= P(1R,3R)

And other similar inequalities.

Simulation of the experiment

- Numerical simulation on computer (10¹⁰ tests done)
- Results for Bell's inequality for different distributions
 - Uniform:
 - 0.027852 + 0.027922 >= 0.027908
 - Non-uniform ex:
 0.050064 + 0.005381 >= 0.044434
 - Independent:
 0.027855 + 0.027609 >= 0.027874

Measuring of electron spin (Stern-Gerlach)

• Electrons are in state:



 $\mathbf{n} = \mathbf{n}' : \text{if } A: \uparrow \text{ then } B: \downarrow$ if $A: \downarrow \text{ then } B: \uparrow$

Probability of $\uparrow \uparrow$ and $\downarrow \downarrow$

- $q(n,n') = 1/2 (1 \cos \vartheta)$
- Correspondence to small envelopes



• Example: $p(1\uparrow,2\uparrow) + p(1\downarrow,2\downarrow) = 1/9 q(\mathbf{n}_1,\mathbf{n}_2)$

Test of Bell's inequalities

- The appropriate Bell's inequality: $q(\mathbf{n}_1, \mathbf{n}_2) + q(\mathbf{n}_2, \mathbf{n}_3) \ge q(\mathbf{n}_1, \mathbf{n}_3)$
- This simplifies to: $1 \ge -\cos(\vartheta_{12} + \vartheta_{23}) + \cos \vartheta_{12} + \cos \vartheta_{23}$
- Maximum violation at:
 - $\vartheta_{_{12}}=\vartheta_{_{23}}=\pi/4$
- Which yields: $1 \ge \sqrt{2}$
- Therefore quantum mechanics violates causality

Summary

- We have studied mathematical model allowing a test for causality
- The model comes in form of inequalities involving probabilities
- We considered probabilities predicted by quantum theory in particular experiment
- We found that the probabilities predicted by quantum mechanics violates these inequalities
- Therefore quantum mechanics violates causality
- This has been verified in experiments to a large extent

Thank You for Your attention