Causality and Quantum Theory

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Abstract:

This miniproject is focused on causality in quantum mechanics and its relation to Bell´s theorem and inequalities. We realised that causality can sometimes violate in quantum theory. Bell´s calculations can be used to estimate whether the quantum system is causal or not.

1 Introduction

According to theory of relativity any massive object or information can´t reach or exceed the speed of light because it would violate causality. In quantum mechanics some systems might violate direct causality because of entanglement.

2 Heisenberg uncertainty principle

We can´t know the position and the momentum in the same time:

$$\Delta x \, \Delta p \geq \hbar/2$$

where x is the position of the particle

where p is the momentum of the particle

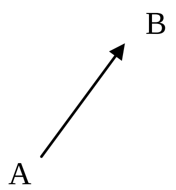
The Heisenberg uncertainty is a consequence of measurement disturbing the state of the particle.

3 EPR paradox

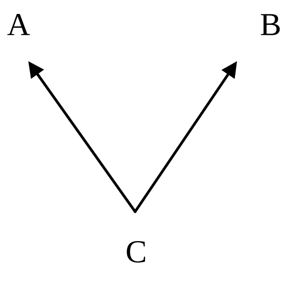
EPR paradox is named after Einstein, Podolsky and Rosen.

Quantum mechanics predicts the existence of correlated particles which violate direct causality.

Causality is correlation between two events.



Direct effect



Particles A and B share common past.
This type is still possible.

4 Bell

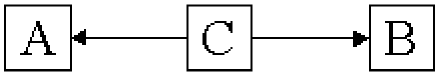
If we assume common cause correlations, equations like

$$p(\boldsymbol{n_1},\boldsymbol{n_2}) + p(\boldsymbol{n_2},\boldsymbol{n_3}) \geq p(\boldsymbol{n_1},\boldsymbol{n_3}),$$

must hold. But quantum mechanics violates (some) of these inequalities.

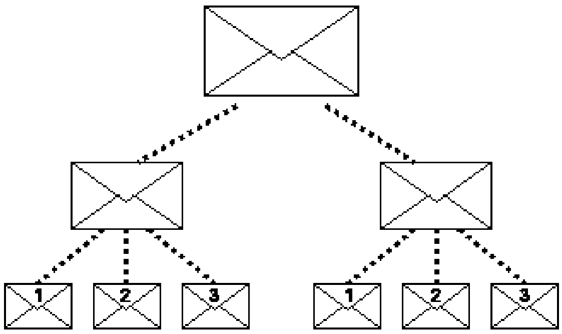
5 Thought experiment

The experiment is helpful to demonstrate an example of Bell´s inequalities. In the experiment are figuring three people: Alice, Bob and Charlie. Charlie can communicate with Alice and



Bob but they can´t communicate with each other.

Charlie prepares big envelopes which each contain two medium envelopes and each medium envelope contains three small envelopes. These are numbered from 1 to 3 and each contains one blue or red coin.



Alice and Bob receive medium envelopes from Charlie. They open these envelopes and choose one of the 3 smallest. If Alice opens her small envelope she realises what Bob has in his own envelope, because she can´t have the same variation like Bob.

They keep repeating this experiment. Each time they write down number of the small envelope and colour of the coins and thus they create a chart.

Alice	Bob
1b	2r
3r	1b
2b	3r
1r	3r
2r	1b

where *r* is red colour
and *b* is blue colour

In three small envelopes of two colours can be four states:

w_a	rrr	bbb
w_b	rbr	brb
w_c	rrb	bbr
w_d	rbb	brr

The probability of Alice choosing the first envelope and finding a red coin and Bob choosing the first envelope and finding a red coin is zero

$$p(1r, 1r) = 0.$$

The probability of Alice choosing the second envelope and finding a red coin and Bob choosing the second envelope and finding a blue coin is given by this equation

$$p(2b, 2r) = 1/18,$$

because there are 3 small envelopes to pick the probability is

$$p(1b, 3r) = (W_c + W_d) / 18.$$

Now we can prove one of the Bell´s inequalities. For example,

$$p(1r, 2r) + p(2r, 3r) \geq p(1r, 3r),$$

leads to

$$1/18(W_b + W_d) + 1/18 (W_b+W_c) \geq 1/18(W_c +W_d)$$

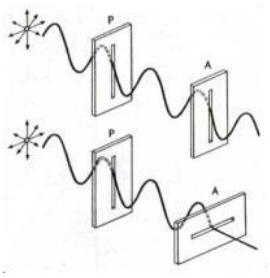
This simplifies to

$$W_b \geq 0,$$

which is always true.

6 Polarization

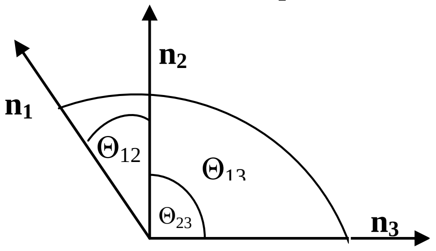
Polarization is an attribute of waves that can oscillate with more than one orientation. Photons also have polarization.



Vertical

Horizontal

Orientation of the polarizer



Quantum mechanics probability:

$$q(\boldsymbol{n_1},\boldsymbol{n_2}) = 2/9 * 1/2\ (1-\cos\theta_{12})$$

Testing Bell´s inequality:

$$q(\boldsymbol{n_1},\boldsymbol{n_2}) + q(\boldsymbol{n_2},\boldsymbol{n_3}) \geq q(\boldsymbol{n_1},\boldsymbol{n_3})$$

Simplified:

$$\cos\theta_{12} + \cos\theta_{23} \leq \cos\theta_{13} + 1$$

When

$$\theta_{12} = 45^\circ$$

$$\theta_{23} = 90^\circ$$

$$\theta_{13} = \theta_{12} + \theta_{23} = 135^\circ$$

the inequality is false: $\sqrt{3} \leq 3/2$

That means Bell´s inequality is violated so causality in relativistic physics is also violated. In the other two arrangements of inequalities the Bell´s inequality is true so the causality in relativistic physics isn´t violated.

7 Conclusion

We have mathematically proved that Bell´s inequalities are sometimes false in quantum systems. We have discovered that causality doesn´t apply every time.

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