Causality and Quantum Theory

Jakub Smrček, Jaroslav Kňap, Václav Otruba
Gymnázium Na Pražačce, Praha
Gymnázium Turnov, Turnov
Gymnázium Jiřího Ortena, Kutná Hora

smrcek.j@seznam.cz
knap.jaroslav@gmail.com
otruba.wasek@gmail.com

Abstract:

This miniproject focused on causality and its relation to Bell's inequalities. Authors came out from the premise, that causality can never be violated in nature. Bell's calculations can be used to verify whether the given system can be considered as causal or not. Authors found out that certain quantum systems can violate the inequalities.

1 Introduction

First of all, authors had to define what the term 'causality' means. By definition, causality is a relationship between two events. First event is called cause and the second is called effect and is a consequence of the first one. Some quantum systems might violate causality. Authors researched this problem in hope that it could be very helpful for man's understanding of the universe.

2 Thought experiment

Authors were introduced with situation where certain rules apply. Let's consider three persons, called Alice, Bob and Cecil. Alice and Bob are not able to communicate with each other but Cecil can communicate with both of them.

Cecil recieves a big envelope. There are two medium envelopes inside of it and each contains three small envelopes. These are numbered from 1 to 3 and each of them contains a single red or a blue disc.

Cecil sends the first medium envelope to Alice and the second one to Bob. They open the envelopes and choose one of the three small ones, discarding the others. Finally, they both open their respective small envelopes, check the colour of the disc inside and note down the results. After that, Cecil hands them another envelopes as this whole process repeats on and on.

A	1R	3R	1B	2B	3R	...
B	1B	2B	2R	1B	3B	...

Notice that the colour of the discs in the envelopes marked with the same number is always opposite - they're anti-correlated. According to this discovery, Alice will know what colour's Bob's disc in his respective envelope.

Thanks to this rule, there are only 4 possible types of big envelopes.

Type	1st medium envelope	2nd medium envelope
A	1R 2R 3R	1B 2B 3B
B	1R 2R 3B	1B 2B 3R
C	1R 2B 3R	1B 2R 3B
D	1R 2B 3B	1B 2R 3R

For each type of the big envelopes is the same probability of being chosen.

w_a for a-type; w_b for b-type; w_c for c-type; w_d for d-type.

According to this rule, the probability that Alice gets a disc of the same colour as Bob's disc in his respective envelope marked with the same number is zero. Mathematically speaking:

p(1R, 1R) = 0 p(2R, 2R) = 0 p(3R, 3R) = 0

p(1B, 1B) = 0 p(2B, 2B) = 0 p(3B, 3B) = 0

It's possible to calculate the probability of all possible combinations, e.g. probability of Alice choosing 1R and Bob choosing 3R is:

$$p(1R, 3R) = 1/3 * 1/3 * (w_b + w_d) * 1/2$$

Bell's inequality comes out of this equation and says:

$$p(1R, 2R) + p(2R, 3R) \geq p(1R, 3R)$$

Leading to:

$$1/9 * (w_c + w_d) * 1/2 + 1/9 * (w_b + w_c) * 1/2 \geq 1/9 * (w_b + w_d) * 1/2$$

We simplify the inequality to:

$$w_c + w_d + w_b + w_c \geq w_b + w_d$$

$$2w_c \geq 0$$

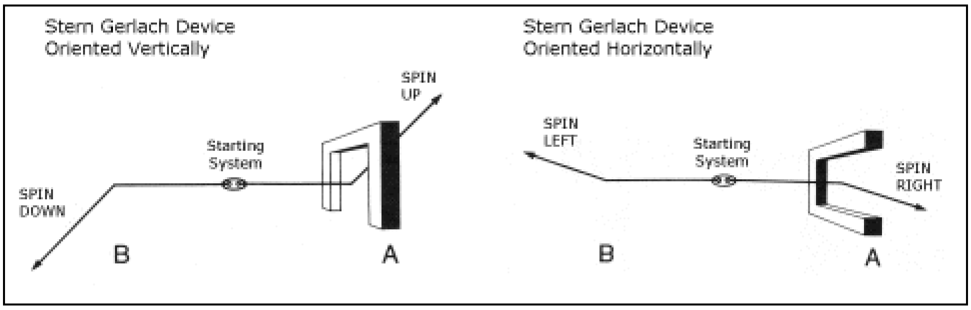
This is always true, because w_c is a probability.

Bell's inequality is always satisfied for any choice of large envelopes, but these inequalities would be mathematically different.

3 Violation of Bell's inequalities in Quantum Mechanical system

Experiment mentioned in previous section can be applied in the quantum realm where it's possible to observe particles instead of discs.

These particles are electrons in special state termed *entangled*. Alice and Bob will obtain these electrons and measure their spins (internal angular momentum of a particle). The measuring apparatus consists of three pairs of magnets:



Alice and Bob can decide which pair of magnets they will use (this corresponds to small envelopes). The spin of electrons can be measured along any axis in space, providing the values $\pm 1/2$. Given the quantum entanglement, spins of electrons are anti-correlated. Thus, when they are measured along the same axis, the measurements with give opposite values.

ϑ_{xy} is the angle between the two axes of magnets undergoing the measurement of spin. The probability of getting the same spin of two different axes is given by this formula:

$$p(x+, y+) = 1/18 * (1 - \cos \vartheta_{xy})$$

It's possible to substitute this formula into Bell's inequality and get:

$$1/18 * (1 - \cos \vartheta_{12}) + 1/18 * (1 - \cos \vartheta_{23}) \geq 1/18 * (1 - \cos \vartheta_{13})$$

$$1 \geq - \cos(\vartheta_{12} + \vartheta_{23}) + \cos \vartheta_{12} + \cos \vartheta_{23}$$

The authors have found the maximum of this function at:

$$\vartheta_{12} = \vartheta_{23} = \pi/4$$

Substituting this equation into the previous one, it gets:

$$1 \geq - \cos(\pi/4 + \pi/4) + \cos \pi/4 + \cos \pi/4$$

$$1 \geq \sqrt{2},$$

which is, of course, wrong, meaning Bell's inequality is violated and that causality in terms of Relativistic physics is also violated.

4 Conclusion

With the help of mathematics, the authors have proven that Bell's inequalities are not always satisfied in quantum system. Thanks to this proof, they discovered that causality does not apply all the time.

Acknowledgements

Authors would like to thank the Czech Technical University in Prague and the Faculty of Nuclear Sciences and Physical Engineering for their support in the Science Week. Authors would also like to thank their supervisor, MSc. Aurél Gábris, PhD, for his support in the field of modern quantum theories and their detailed explanation, regarding the topic. The authors also wish to explicitly thank Ing. Vojtěch Svoboda for the organization of the whole Science Week and for his enthusiasm in helping to set our project on the right path.

References:

[1] EINSTEIN, A. – PODOLSKY, B. – ROSEN, N.: *Can Quantum-Mechanical Desription of Physical Reality Be Considered Complete?* Physical Rev. 47, 1935, 777–780

[2] BELL, J. S.: *Speakable and Unspeakable in Quantum Mechanics* Cambridge University Press, 1988

[3] CLAUSER, J. F. – SHIMONY, A.: *Bell's theorem: Experimental tests and implications* Rep. Progr. Phys. 41, 1881 (1978)