Causality and Quantum Theory

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Abstract:

"God does not play dice." [1] Or does he? Does quantum theory really describe nature? Is there true randomness at the subatomic level, or do particles know what to do, controlled by variables beyond our current understanding and means of detection? Can we find an alternative theory that would get rid of those pesky probabilities? In two simple experiments, involving colourful envelopes and photons that had more in common than one would think, we looked for a way to evaluate the accuracy of quantum theory as compared to its alternatives, and found that the violation of Bell's inequalities within quantum theory may well be the key.

1 Introduction

On macroscopic scales, the behaviour of objects is governed by classical physics. For reasons that aren't yet fully clear, these laws fail to accurately represent reality on a microscopic scale. At this point, the best theory that explains events on the atomic and subatomic level is quantum theory. Quantum theory describes the microscopic world in terms of probabilities. For instance, suppose a photon meeting an obstacle, where it can either be transmitted or reflected. Quantum mechanics can determine the respective probabilities of the photon being absorbed or being reflected, but it cannot absolutely predict what will happen. In short, quantum mechanics is not deterministic.

This poses the question of whether quantum theory is complete: are events at the atomic and subatomic level accurately described using probabilities, or are there underlying causes underneath the apparent randomness, which we simply cannot detect yet?

An alternative to quantum mechanics could be a theory that operates with hidden variables: the idea that in addition to its normal parameters such as wavelength, vector of propagation, etc., a particle has another variable or a set of them that determines its behaviour.

In our project, we will try to see how we can decide between quantum theory and its deterministic alternatives. We will find where the theories differ and from there, we can go on to find out which option describes nature better.

2 Violation of Bell's inequalities

In order to determine the difference between quantum theory and its alternatives, we chose to use Bell's inequalities [2]. First, we modelled a situation using a simple setup with envelopes and coloured pieces of paper, then we translated the experiment into the quantum world.
When a correlation is observed in a set of data, it can be assumed that there is some relation between the variables. One way to explain a correlation is a cause-effect relationship, where one variable has a direct effect on another. The other option is a common cause relationship, where the two values are affected by the same separate variable.
A way to mathematically determine the type of correlation is by using Bell's inequalities, which compare the probabilities of events. These inequalities are always satisfied for events with a common cause, but may be violated for non-common cause correlations.

We used the following formula:

$P(A,B) + P(B,C) \geq P(A,C)$

2.1 Envelope experiment
We used three colours (denoted varieties below) of Post-It notes: green (1), orange (2), and pink (3). We took two notes of each colour, marked one of the pair in red (R) and the other in blue (B), and separated the pairs into two envelopes, so that each contained one note of each variety. The combination of red and blue markings in each envelope was random. This way, we prepared several pairs of envelopes and placed each in a larger envelope.
We then randomly chose a large envelope, drew one note from each smaller one within, and wrote down its variety and colour.

Data set:
3B 2B
3R 1R
1R 1B
3R 3B
2B 1B
2B 1R

It was found that, because of the way the envelopes were prepared, a correlation could be observed: if two notes of the same variety were drawn, one had to be blue and the other had to be red. There is a common cause, because the envelopes were prepared in advance.
We went on to test whether this correlation satisfied Bell's inequality.
First, the probabilities of getting a certain combination (for example, 3B-2B) were determined.
We found that there were four types of large envelopes:

	1	2	3	1	2	3
a	B	B	B	R	R	R
b	B	B	R	R	R	B
c	B	R	B	R	B	R
d	B	R	R	R	B	B

We denoted the probabilities of choosing each W_a, W_b, W_c, W_d. To these, we can apply general rules of probability:

$W_a + W_b + W_c + W_d = 1$

$0 \leq W_{a,b,c,d} \leq 1$

In these terms, the probabilities of each combination could be expressed. For example, in the case of 3B-2B:
This combination could only be found in envelope type b OR c. Therefore:

$P(3B,2B) = P(3B) \cdot P(2B) \cdot (W_b + W_c) = 1/3 \cdot 1/3 \cdot (W_b + W_c) = 1/9(W_b+W_c)$

Likewise, the probability of combination 1R-1B can be expressed, knowing that this combination exists in every type of envelope:

$P(1R,1B) = P(1R) \cdot P(1B) \cdot (W_a+W_b+W_c+W_d) = 1/3 \cdot 1/3 \cdot 1 = 1/9$

Using these, we attempted to verify Bell's inequality.

$P(3B,2B) + P(2B,1B) \geq P(3B,1B)$
$1/9(W_b+W_c) + 1/9(W_c+W_d) \geq 1/9(W_b+W_d)$
$W_c \geq 0$

This result follows the rule established above: $W_{a,b,c,d} \geq 0$. Bell's inequality was satisfied.

2.2 Photon experiment

In the next step, we attempted to translate our experiment into the quantum world and see if the outcome would be similar, using the example of a photon hitting a polarization beam splitter (PBS). When polarized light hits the PBS, part of the beam is transmitted and part is reflected.

For transmitted light: $I = \cos^2\varphi \cdot I_0$
For reflected light: $I = \sin^2\varphi \cdot I_0$
where φ is the angle between the splitter and the vector of polarization.

When a single photon hits the PBS, it cannot split; it must be either transmitted or reflected. Within quantum theory, the photon can be described in terms of probability that it will be transmitted, and probability that it will be reflected.
Our experiment involves a source which emits two photons at the same time. When a beam splitter is placed in the way of each, the probability that the two photons will have opposite polarization can be expressed as:

$P(\varphi_A,\varphi_B) = \cos^2(\varphi_A-\varphi_B)$

where φ_A, φ_B are the angles of the splitters.
The probability that they will have the same polarization can be expressed as:

$P(\varphi_A,\varphi_B) = \sin^2(\varphi_A-\varphi_B)$

Therefore, the probability of only vertical or only horizontal polarization is:

$P(\varphi_A,\varphi_B) = 1/2\sin^2(\varphi_A-\varphi_B)$

These relationships imply that, if the splitters are at the same angle, the photons will always have opposite polarizations, one horizontal and one vertical. This must be a common cause correlation, because there is no communication between the photons after they are emitted, ruling out a cause-effect relationship.
This situation is analogous to our envelope experiment, where the colours red and blue represent vertical and horizontal polarization, and the colours green (1), orange (2) and pink (3) represent splitter angles.
We can translate the experiment as such:
Let the colours 1, 2, 3 be angles $\varphi_1 = 30^\circ$, $\varphi_2 = 60^\circ$, $\varphi_3 = 90^\circ$.
Let the colour blue (B) be horizontal polarization (H), and red (R) be vertical (V).
Finally, the probabilities can be plugged into Bell's inequality, using the one formulated during the envelope experiment.

$P(3H,2H) + P(2H,1H) \geq P(3H,1H)$
$1/9 \cdot 1/2 \cdot \sin^2(90^\circ-60^\circ) + 1/9 \cdot 1/2 \cdot \sin^2(60^\circ-30^\circ) \geq 1/9 \cdot 1/2 \cdot \sin^2(90^\circ-30^\circ)$
$1/72 + 1/72 \geq 1/24$
$1/36 \geq 1/24$

In this case, Bell's inequality was violated.

3 Conclusion

We have found that, within the framework of quantum theory, Bell's inequalities can be violated, while in alternative theories such as the hidden variable theory, they cannot. As such, Bell's inequalities are a mathematical tool that can be used to distinguish between quantum theory and its alternatives. Experimental results can then decide which theory describes nature better [3]. To date, experiments have violated Bell's inequalities and therefore supported quantum theory. However, the question whether quantum theory is complete – whether "God plays dice" or not – remains open to further investigation.

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